GEOMETRICAL AND PHENOMENOLOGICAL TORSIONS IN RELATIVISTIC PHYSICS

G.I.Shipov

Abstract

The Ricci and Cartan torsions are compared. Their common properties and distinctions are revealed. It is shown that Ricci torsion determines curvature and torsion in Frenet's equations and consequently the very torsion, instead of the Cartan one, can be connected with spin properties of matter. The theorems showing that in flat and curved spaces it is possible to present Frenet's curves as a first kind geodesic lines of space of absolute parallelism are proved . The connection between fields and forces of inertia and torsion of geometry of absolute parallelism (Ricci torsion) is established.

On the basis of Frenet's equations the structure of radiation friction force in equations of charge motion possessing a spin is investigated. It is shown, that there is a radiation force connected with the charge spin, which is responsible for torsion component of electromagnetic field. The theoretical evaluation of this force magnitude is given. It is concluded, that torsion interactions are weaker than electromagnetic ones, but stronger than gravitational ones.

1 Frenet's equations

In the middle of the last century French mathematician F. Frenet has written famous equations, describing motion of orientable point[1] along arbitrary curve $\mathbf{x} = \mathbf{x}(\mathbf{s})$, where s – the length of an arc.

Frenet's equations are written for normalized on unit orthogonal vectors $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$ with the beginning in a point M (fig.1). They are as follows [1]

$$\frac{d\mathbf{e}_1}{ds} = \kappa(s)\mathbf{e}_2 \ , \tag{1}$$

$$\frac{d\mathbf{e}_2}{ds} = -\kappa(s)\mathbf{e}_1 \ + \chi(s)\mathbf{e}_3 \ , \tag{2}$$

$$\frac{d\mathbf{e}_3}{ds} = -\chi(s)\mathbf{e}_2 \ , \tag{3}$$

where $\kappa(s)$ - curvature of the curve, $\chi(s)$ - torsion of the curve.

Frenet was the first who has shown that arbitrary curve is generally determined by two parameters - curvature $\kappa(s)$ and torsion $\chi(s)$.

Unit vector $\mathbf{e}_1$ is choused as tangent to the curve at point M.

$$\frac{d\mathbf{x}}{ds} = \mathbf{e}_1 \ , \quad \left|\frac{d\mathbf{x}}{ds}\right| = 1 \tag{4}$$

[1] An orientable point is understood as triad, formed by three unit orthogonal vectors.

Unit vector $\mathbf{e}_2$ is directed along main normal and the binormal vector $\mathbf{e}_3$ is determined as follows

$$\mathbf{e}_3 = [\mathbf{e}_1 \mathbf{e}_2].$$

Differentiating Frenet's equations (1) and (2) on s and using orthogonality conditions of the triad vectors, we shall get the equations

$$\frac{d^2\mathbf{x}}{ds^2} = \kappa(s)\mathbf{e}_2 \ , \tag{5}$$

$$\frac{d^3\mathbf{x}}{ds^3} = \frac{d\kappa(s)}{ds}\mathbf{e}_2 - \kappa^2(s)\mathbf{e}_1 + \kappa(s)\chi(s)\mathbf{e}_3 \ , \tag{6}$$

describing motion of the triad initial point (motion of point M).

During infinitesimal displacement of point M along the curve the triad vectors simultaneously change their orientation in space. For description of the change it is convenient to introduce angular coordinates ψ, φ, ϕ. Expressing, for example, the components of tangent vector $\mathbf{e}_1$ through angular variables, we have

$$\frac{dx}{ds} = cos\varphi \ cos\psi - sin\varphi \ sin\psi \ cos\phi, \tag{7}$$

$$\frac{dy}{ds} = sin\varphi \ cos\psi + cos\varphi \ sin\psi \ cos\phi, \tag{8}$$

$$\frac{dz}{ds} = sin\psi \ sin\phi. \tag{9}$$

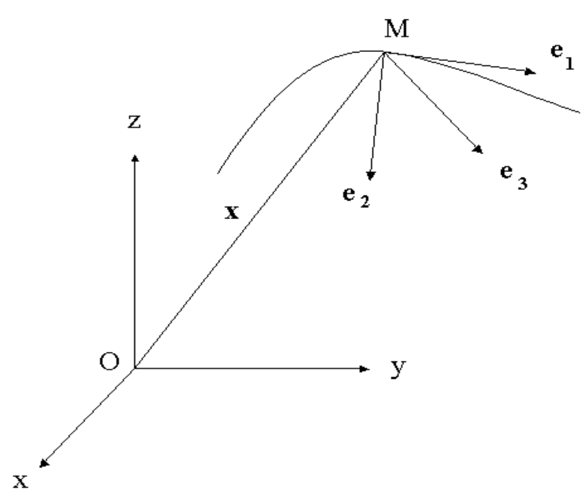

Figure 1: Trajectory of an orientable point

Differentiating these equations and excluding from them translational coordinates, we get "rotational equations of motion" as follows

$$\frac{d\varphi}{ds} = \chi \frac{sin\psi}{sin\phi}, \tag{10}$$

$$\frac{d\psi}{ds} = \kappa - \chi\ sin\psi\ ctg\phi, \tag{11}$$

$$\frac{d\phi}{ds} = \chi\ cos\psi. \tag{12}$$

2 Connection of $\kappa(s)$ and $\chi(s)$ with Ricci torsion

Let us investigate the issue of geometry structure, in which Frene's curves are geodesic lines.

Statement 1. Curvature κ and torsion χ are independent components of Ricci rotation coefficients.

Proof. Let's consider six-dimensional manifold of coordinates $x_1, x_2, x_3, \varphi_1, \varphi_2, \varphi_3$. It is convenient to present it as a vector bundle[2] with the base formed by translational coordinates x_1, x_2, x_3 (let it be Cartesian coordinates) and fibre, specified at each point x_a $(a = 1, 2, 3)$ by three orthonormalized Frenet's reference vectors

$$\mathbf{e}_A, \qquad\qquad A = 1, 2, 3, \tag{13}$$

where A means number of the reference vector.

According to Euler's theorem, an infinitesimal rotations around the three axes of reference vector (13) is equivalent to one rotation with angle $\mathbf{d}\boldsymbol{\chi}$ around a definite axis passing through the origin of the axis O. It is possible to define the infinitesimal rotation as

$$\mathbf{d}\boldsymbol{\chi} = d\chi \mathbf{e}_\chi,$$

where vector $\mathbf{e}_\chi$ is directed along instantaneous rotation axis of reference system. This direction is selected so that, if one looks from the end of the vector $\mathbf{e}_\chi$ e at a fixed point O, then the rotation is made counter-clockwise (right-hand reference system).

An infinitesimal rotation of Frenet's reference vectors $\mathbf{e}_\chi$ upon rotation $\mathbf{d}\boldsymbol{\chi}$ has the form

$$d\mathbf{e}_A = [\mathbf{d}\boldsymbol{\chi}\mathbf{e}_A]. \tag{14}$$

If we divide (14) by ds, then we shall get

$$\frac{d\mathbf{e}_A}{ds} = [\frac{\mathbf{d}\boldsymbol{\chi}}{ds}\mathbf{e}_A] = [\boldsymbol{\omega}, \mathbf{e}_A], \tag{15}$$

where $\boldsymbol{\omega} = \mathbf{d}\boldsymbol{\chi}/ds$ - three-dimensional angular velocity of Frenet's triad with respect to the instantaneous axis. Writing down Frenet's reference vectors in the form

[2] *The term vector bundle is excepted in mathematics.*

$$a) \qquad e^A{}_\alpha e^\alpha_B = \delta^A_B = \begin{cases} 1 & A = B \\ 0 & A \neq B \end{cases}, \tag{16}$$

$$b) \qquad e^A{}_\alpha e^\beta{}_A = \delta_\alpha{}^\beta[] = \begin{cases} 1 & \alpha = \beta \\ 0 & \alpha \neq \beta \end{cases},$$

$$A, B ... = 1, 2, 3, \qquad\qquad \alpha, \delta, \beta = 1, 2, 3,$$

where $\alpha, \delta, \beta...$ – vector indices, and $A, B...$ – triad indices; it is possible to write down relations (14) and (15) as follows

$$de^A{}_\alpha = d\chi^\beta{}_\alpha e^A{}_\beta \quad or \quad d\chi^\beta{}_\alpha = T\,{}^\beta_{\alpha\gamma} dx^\gamma, \tag{17}$$

$$\frac{de^A{}_\alpha}{ds} = \frac{d\chi^\beta{}_\alpha}{ds} e^A{}_\beta \quad or \quad \frac{de^A{}_\alpha}{ds} = T^\beta_{\alpha\gamma}\frac{dx^\gamma}{ds} e^A{}_\beta, \tag{18}$$

where we have defined the designation

$$T^\alpha_{\beta\gamma} = e^\alpha_A e^A{}_{\beta,\gamma} = -e^A_\beta e^\alpha{}_{A,\gamma}, \qquad ,\gamma = \frac{\partial}{\partial x^\gamma}. \tag{19}$$

The quantities (19) were first introduced by G.Ricci [2] and since then they have been called Ricci rotation coefficients. Using the orthogonality conditions (16) and the rule of transformation to local indices

$$T^A_{B\gamma} = e^A{}_\alpha T^\alpha_{\beta\gamma} e^\beta{}_B \ ,$$

let's rewrite equations (18) in local indices

$$\frac{de^A{}_\alpha}{ds} = T^A{}_{B\gamma}\frac{dx^\gamma}{ds} e^B{}_\alpha \ . \tag{20}$$

Let's chose vectors $e^{(1)}{}_\alpha, e^{(2)}{}_\alpha$ and $e^{(3)}{}_\alpha$ so, that they coincide with Frenet's vectors, and thus the vector $e^{(1)}{}_\alpha$ satisfies conditions (4). Then the equations (20) become the well-known Frenet's equations (1-3), in which

$$\kappa(s) = T^{(1)}{}_{(2)\gamma}\frac{dx^\gamma}{ds} \ , \quad \chi(s) = T^{(2)}{}_{(3)\gamma}\frac{dx^\gamma}{ds} \ . \tag{21}$$

While deducing (1-3) from (20), we used the following relations

$$\frac{dx^\gamma}{ds} = e^\gamma{}_{(1)} \ \ and \ \ e^\gamma_{(1)} e^{(1)}_\gamma = 1 \ .$$

From the relations (21) it is clear, that in Frenet's equations curvature and torsion are expressed through components of Ricci rotation coefficients (19), that proves the statement 1.

The Ricci rotation coefficients are the part of the connection of absolute parallelism [3] and have an anti-symmetry on the two lower indices

$$T^{\alpha}{}_{[\beta\gamma]} = -\Omega^{..\alpha}_{\beta\gamma},$$

$$\Omega^{..\alpha}_{\beta\gamma} = -\frac{1}{2} e^{\alpha}_{A}(e^{A}{}_{\beta,\gamma} - e^{A}{}_{\gamma,\beta}), \tag{22}$$

which it is possible to call *Ricci torsion*.

3 Frenet's curves as geodesic of the geometry of absolute parallelism

Let's note, that the curvature and torsion of Frenet's curve would be more correctly called the first and second torsion, as they are both expressed through components of Ricci torsion (22).

Until to this time we have used as translational coordinates Cartesian ones. Now let's now from Cartesian coordinates to arbitrary curvilinear ones.

In general case when we have arbitrary curvilinear translational coordinates the metric tensor of space is represented in the following form

$$\overset{0}{g}_{\alpha\beta} = \eta_{AB} e^{A}{}_{\alpha} e^{b}{}_{\beta}, \qquad \eta_{AB} = \eta^{AB} = \mathrm{diag}(1,1,1),$$

and translational interval as

$$ds^2 = \overset{0}{g}_{\alpha\beta}\, dx^{\alpha} dx^{\beta} = \eta_{AB} e^{A}{}_{\alpha} e^{b}{}_{\beta} dx^{\alpha} dx^{\beta} \ . \tag{23}$$

In arbitrary translational coordinates the total connection of the space can be written down as

$$\Delta^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\beta\gamma} + T^{\alpha}{}_{\beta\gamma} = e^{\alpha}_{A} e^{A}{}_{\beta,\gamma},$$

where

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} \overset{0}{g}{}^{\alpha\eta} (\overset{0}{g}_{\beta\eta,\gamma} + \overset{0}{g}_{\gamma\eta,\beta} - \overset{0}{g}_{\beta\gamma,\eta}) \tag{24}$$

– Christoffel's symbols,

$$T^{\alpha}{}_{\beta\gamma} = -\Omega^{..\alpha}_{\beta\gamma} + \overset{0}{g}{}^{\alpha\eta} (\overset{0}{g}_{\beta\rho}\, \Omega^{..\rho}_{\eta\gamma} + \overset{0}{g}_{\gamma\rho}\, \Omega^{..\rho}_{\eta\beta}) \tag{25}$$

– Ricci's rotation coefficients, and $\Omega^{..\alpha}_{\beta\gamma}$ is defined according to (22). This tensor is distinct from zero, when, while describing dynamics of rotational motion, angular nonholonomic coordinates $\varphi_1, \varphi_2, \varphi_3$ are used.

Now equality (17) will be written as follows

$$d\chi_{\beta\alpha} = \Delta^{\alpha}{}_{\beta\gamma} dx^{\gamma}, \tag{26}$$

where quantities

$$\Delta^{\alpha}{}_{\beta\gamma} = e^{\alpha}_{A} e^{A}{}_{\beta,\gamma} = -e^{A}_{\beta} e^{\alpha}{}_{A,\gamma}. \tag{27}$$

represent the local connection of affine space. Like any connection it has nontensor law of transformation with respect to transformations of translational coordinates

$$\Delta^{\gamma'}{}_{\beta'\alpha'} = \frac{\partial^2 x^{\gamma}}{\partial x^{\alpha'}\partial x^{\beta'}}\frac{\partial x^{\gamma'}}{\partial x^{\gamma}} + \frac{\partial x^{\alpha}}{\partial x^{\alpha'}}\frac{\partial x^{\beta}}{\partial x^{\beta'}}\frac{\partial x^{\gamma'}}{\partial x^{\gamma}}\Delta^{\gamma}{}_{\beta\alpha}.$$

If now we shall form curvature tensor with the help of connection (27), then it appears to be equal to zero [3]

$$S^{\alpha}_{\beta\gamma\eta} = 2\Delta^{\alpha}{}_{\beta[\eta,\gamma]} + 2\Delta^{\alpha}{}_{\rho[\gamma}\Delta^{\rho}_{|\beta|\eta]} = 0.$$

By the definition, space with zero curvature tensor is called a space of absolute parallelism, and the relation (27) defined the connection of absolute parallelism.
Euclidean space is a particular case of a space of absolute parallelism. Really, from the formula (27) it is clear that when the rotation is absent ($d\chi^{\alpha}{}_{\beta} = 0, \quad dx^{\gamma} \neq 0$), then the connection $\Delta^{\alpha}{}_{\beta\gamma}$ becomes zero, thus the space of absolute parallelism becomes Euclidean.

Statement 2. Frenet's equations are equivalent to the geodesic equations of the first kind (the shortest)of the geometry of absolute parallelism.

$$\frac{d^2x^{\alpha}}{ds^2} = -\Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{ds}\frac{dx^{\gamma}}{ds} - T^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{ds}\frac{dx^{\gamma}}{ds}. \tag{28}$$

Proof. In arbitrary coordinates equations (18) will be written in the form

$$a)\ \ \frac{de^A_{\alpha}}{ds} = \Gamma^{\beta}{}_{\alpha\gamma}\frac{dx^{\gamma}}{ds}e^A{}_{\beta} + T^{\beta}{}_{\alpha\gamma}\frac{dx^{\gamma}}{ds}e^A{}_{\beta}, \quad or \quad b)\ \ \frac{de^{\alpha}_A}{ds} = -\Gamma^{\alpha}{}_{\beta\gamma}\frac{dx^{\gamma}}{ds}e^{\beta}{}_A - T^{\alpha}{}_{\beta\gamma}\frac{dx^{\gamma}}{ds}e^{\beta}{}_A, \tag{29}$$

Since in Frenet's equations the vector $e^{\alpha}{}_{(1)} = dx^{\alpha}/ds$, then, substituting the relation into equations (29 b), we obtain the geodesic equations (28).

4 Phenomenological Cartan torsion

The above proved statements allow to assert that it is possible to present any curve, defined in space with the flat metric (23), as a geodesics of the space of absolute parallelism with equation (28). It is direct way to geometrization of physical equations, since any curve can be considered as trajectory of a particle, which moves in some physical field.
We shall name Ricci torsion as geometric one, since it is defined through derivatives of vectors of Frenet triad and is geometrically interpreted as rotation of the triad during its motion along the curve. As it was shown earlier, the geometric torsion is included in the structure of connection (27) of absolute parallelism geometry.
Alongside with geometric torsion it is possible to introduce phenomenological Cartan torsion [4], which has the same symmetry properties, as Ricci torsion, but unlike the latter is not connected to Frenet triad rotation, since it does not depend on its vectors.
In the beginning proceeding from group properties of space of absolute parallelism with constant curvature E.Cartan and J. Schouten [5, 6] have introduced connection (27), in which components of Ricci rotation coefficients are constants.
The essence of E.Cartan and J. Schouten approach consists in the following. Let on n-dimensional differentiable manifold M with coordinates $x^1, \dots, x^n$ the field of n contravariant vectors is given

$$\xi^j_a = \xi^j_a(x^k), \tag{30}$$

where

$$a, b, c \ldots = 1 \ldots n$$

are vector indices, and

$$i, j, k \ldots = 1 \ldots n$$

– coordinate indices.
Suppose that

$$\det(\xi^j_a) \neq 0$$

and the functions ξ^j_a satisfy the equations

$$\xi^j_a \xi^k{}_{b,j} - \xi^i_b \xi^k{}_{a,j} = -C^{..f}_{ab}\,\xi^k_f,$$

in which constants $C^{..f}_{ab}$ have the following properties:

$$C^{..f}_{ab} = -C^{..f}_{ba}, \tag{31}$$

$$C^{..a}_{fb} C^{..f}_{cd} + C^{..a}_{fc} C^{..f}_{db} + C^{..a}_{fd} C^{..f}_{bc} = 0. \tag{32}$$

Then we can say that we have n-parametric simple transitive group (group T_n), operating in the manifold, such that $C^{..f}_{ab}$ are structural constants of the group that obey the Jacobi identity (32). The vector field ξ^j_b is said to be infinitesimal generators of the group.
Let now the basis $e^j{}_a$, defined in each point of the manifold M, satisfy the condition

$$\det(e^j{}_a) \neq 0.$$

If we suppose that

$$e^j{}_a(x^k_0) = \xi^j_a(x^k_0),$$

where x^k_0 are the coordinates of some arbitrary point P, then we have for the functions $e^j{}_a(x^k_0)$ the equations

$$e^j{}_a e^k{}_{b,j} - e^j{}_b e^k{}_{a,j} = -C^{..f}_{ab}\, e^k{}_f. \tag{33}$$

It follows from the normalization condition for the basis

$$e^a{}_i e^j{}_a = \delta^j_i, \quad e^a{}_i e^i{}_b = \delta^a_b \tag{34}$$

from equality (33) that

$$C^{..i}_{jk} = 2e^i{}_a e^a{}_{[k,j]} = e^i{}_a C^{..a}_{bc} e^b_j e^c_k. \tag{35}$$

Comparing (35)and (22), we see that

$$\Omega^{..i}_{jk} = \frac{1}{2} C^{..i}_{jk},$$

i.e. all the components of Ricci torsion of homogeneous space of absolute parallelism are constant.

It is easily seen that

$$\Delta^k_{[ij]} = -\Omega^{..k}_{ij} = T^k_{[ij]} = -\frac{1}{2}C^{..i}_{jk}.$$

Since the constants don't depend on coordinates or any other variables, then E.Cartan and J. Schouten introduced the connection with phenomenological torsion [5, 6], which is the tensor S^i_{jk} with symmetry properties as Ricci torsion. Thus obtained geometric structure is called Riemann- Cartan geometry with connection

$$\bar{\Gamma}_{ijk} = \Gamma_{ijk} + (S_{ijk} - S_{jki} - S_{kij}), \tag{36}$$

where Γ_{ijk} - Christoffel symbols, and S_{ijk} - tensor of Cartan torsion. It is possible to state with confidence that the Riemann - Cartan geometry has appeared as a result of development of the geometry of absolute parallelism.

5 Common properties and distinctions between Ricci and Cartan torsions

Phenomenological Cartan torsion $S^i{}_{jk}$ has many common properties with geometric Ricci torsion $\Omega^{..i}_{jk}$, however between them there is also an essential difference.

5.1 Common properties of Ricci and Cartan torsions

.

- Identical number of independent components and inferior indices antisymmetry

$$\Omega^{..i}_{jk} = -\Omega^{..i}_{kj}, \qquad S^{..i}_{jk} = -S^{..i}_{kj}. \tag{37}$$

- Tensor transformation law with respect to translational coordinates

$$\Omega^{..i'}_{j'k'} = \Omega^{..i}_{jk}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^k}{\partial x^{k'}}\frac{\partial x^{i'}}{\partial x^i}, \qquad S^{..i'}_{j'k'} = S^{..i}_{jk}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^k}{\partial x^{k'}}\frac{\partial x^{i'}}{\partial x^i}. \tag{38}$$

- The contorsion tensor is formed by the same way

$$T^i{}_{jk} = -\Omega^{..i}_{jk} + g^{im}(g_{js}\Omega^{..s}_{mk} + g_{ks}\Omega^{..s}_{mj}), \quad K^i{}_{jk} = -S^{..i}_{jk} + g^{im}(g_{js}S^{..s}_{mk} + g_{ks}S^{..s}_{mj}) \tag{39}$$

with symmetry properties

$$T_{ijk} = -T_{jik}, \qquad K_{ijk} = -K_{jik}. \tag{40}$$

- The torsions can be decomposed into irreducible parts in the same manner

$$\Omega^i_{.jk} = \frac{2}{3}\delta^i_{[k}\Omega_{j]} + \frac{1}{3}\varepsilon^n_{jks}\hat{\Omega}^s + \overline{\Omega}^i_{.jk}, \quad S^i_{.jk} = \frac{2}{3}\delta^i_{[k}S_{j]} + \frac{1}{3}\varepsilon^n_{jks}\hat{S}^s + \overline{S}^i_{.jk}, \tag{41}$$

where

$$\Omega^i_{.jk} = g^{im}g_{ks}\Omega^{..s}_{mj}, \qquad S^i_{.jk} = g^{im}g_{ks}S^{..s}_{mj}$$

and

– vectors

$$\Omega_j = \Omega^i_{.ji}, \qquad S_j = S^i_{.ji}, \tag{42}$$

– pseudovectors

$$\hat{\Omega}_j = \frac{1}{2}\varepsilon_{jins}\Omega^{ins}, \qquad \hat{S}_j = \frac{1}{2}\varepsilon_{jins}S^{ins}, \tag{43}$$

– traceless parts of torsion

$$\overline{\Omega}^s_{.js} = 0, \quad \overline{\Omega}_{ijs} + \overline{\Omega}_{jsi} + \overline{\Omega}_{sij} = 0, \quad \overline{S}^s_{.js} = 0, \quad \overline{S}_{ijs} + \overline{S}_{jsi} + \overline{S}_{sij} = 0. \tag{44}$$

5.2 Distinctions between Ricci and Cartan torsions

Let's mark the following differences between Ricci and Cartan torsion.

- Ricci torsion $\Omega^{..i}_{jk}$ determines change of orientation of frame vectors (i.e. it depends on angular coordinates)

$$\Omega^{..i}_{jk} = -\frac{1}{2}e^i_a(e^a{}_{j,k} - e^a{}_{k,j}), \quad i, j, i... = 0, 1, 2, 3, \quad a, b, c... = 0, 1, 2, 3, \tag{45}$$

  and Cartan torsion $S^{..i}_{jk}$ depends only on translational coordinates x^i.

- In space with four translational coordinates the torsion $\Omega^{..i}_{jk}$ is defined on 10-dimensional manifold, whereas torsion $S^{..i}_{jk}$ only on four-dimensional one.

- Torsion $\Omega^{..i}_{jk}$ determines additional (to the translational Riemannian metric) rotational metric [3]

$$d\tau^2 = T^a{}_{bk}T^b{}_{an}dx^k dx^n, \tag{46}$$

  and torsion $S^{..i}_{jk}$ does not.

- Torsion $\Omega^{..i}_{jk}$ allows to present any curve in Riemannian space as geodesics of space of absolute parallelism. In space with four translational coordinates torsion $\Omega^{..i}_{jk}$ defines three optical parameters: expansion θ, rotation ω and shift σ. These parameters allow to give a kinematic interpretation to components of torsion $\Omega^{..i}_{jk}$, in particular to connect torsion property of a matter with optical parameter of rotation ω. Cartan torsion has not such property.

- Experiments with electrotorsion Akimov's generators [7] find their explanation through Ricci torsion $\Omega^{..i}_{jk}$, but not through Cartan torsion $S^{..i}_{jk}$.

6 Kinematic interpretation of curvature and torsion in Frenet's equations

Let's consider an orientable material point[3] , which moves along arbitrary curve

$$\mathbf{x} = \mathbf{x}(\mathbf{s}).$$

Let this curve is described by Frenet's equations (1-6). To find out a physical meaning of curvature and torsion, let's consider two important limit cases; a) $\kappa \neq 0$, $\chi = 0$ and b) $\kappa = 0$, $\chi \neq 0$.

6.1 Curves with $\kappa \neq 0$, $\chi = 0$

In this case the equations (1-6) get the following form

$$a)\ \ \frac{d\mathbf{e}_1}{ds} = \kappa(s)\mathbf{e}_2\ , \quad b)\ \ \frac{d\mathbf{e}_2}{ds} = -\kappa(s)\mathbf{e}_1\ , \quad c)\ \ \frac{d\mathbf{e}_3}{ds} = 0\ , \tag{47}$$

$$a)\ \ \frac{d^2\mathbf{x}}{ds^2} = \kappa(s)\mathbf{e}_2\ , \quad b)\ \ \frac{d^3\mathbf{x}}{ds^3} = \frac{\kappa(s)}{ds}\mathbf{e}_2 - \kappa^2(s)\mathbf{e}_1\ . \tag{48}$$

Curves, described by these equations, are "flat", since all their points lie in the same plane. It is known from mechanics that the orbital moment conservation law is executed when particles move in fields with central symmetry along trajectories lying in the same plane. The derivative

$$\frac{d\mathbf{x}}{dt} = \mathbf{v},$$

where t - time, which defines the velocity of material point (velocity of the origin of Frenet's triad) along trajectory. This relation can be written down in the following form

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \frac{d\mathbf{x}}{ds}\frac{ds}{dt} = \mathbf{e}_1\frac{ds}{dt}. \tag{49}$$

Since $\mathbf{e}_1$ - the unit vector, then

$$|\mathbf{v}| = \frac{ds}{dt} = v.$$

The total acceleration $\mathbf{w} = d^2\mathbf{x}/dt^2$ will be written down as follows

$$\mathbf{w} = \frac{d^2\mathbf{x}}{ds^2}\left(\frac{ds}{dt}\right)^2 + \mathbf{e}_1\frac{d^2s}{dt^2}. \tag{50}$$

Using Frenet's equations, we shall obtain from (50)

$$\mathbf{w} = \mathbf{e}_2\kappa\left(\frac{ds}{dt}\right)^2 + \mathbf{e}_1\frac{d^2s}{dt^2} = \mathbf{e}_2\kappa v^2 + \mathbf{e}_1\frac{dv}{dt}. \tag{51}$$

[3]Orientable mass point is an orientable point possessing mass m and moment of inertia J.

From relation (51) one can see that the acceleration is decomposed into sum of two terms, one of which is tangent and is called *tangential acceleration*

$$\mathbf{w}_\tau = \mathbf{e}_1 \frac{dv}{dt},$$

and another is directed along the main normal

$$\mathbf{w}_n = \mathbf{e}_2 \kappa v^2$$

and is called *normal acceleration.* From the last relation one can see that *the curvature of the curve defines normal acceleration of orientable point.*

6.2 Curves with $\kappa = 0$, $\chi \neq 0$

In this case equations (1-6) will be written in the form

$$\frac{d\mathbf{e}_1}{ds} = 0 \ , \quad \frac{d\mathbf{e}_2}{ds} = \chi(s)\mathbf{e}_3 \ , \quad \frac{d\mathbf{e}_3}{ds} = -\chi(s)\mathbf{e}_2 \ , \tag{52}$$

$$\frac{d^2\mathbf{x}}{ds^2} = 0 \ , \quad \frac{d^3\mathbf{x}}{ds^3} = 0 \ . \tag{53}$$

Since equations (53) of this system describe a motion of point M (motion of origin of Frenet's triad), then we see that in this case the curve, along which the tangent vector $\mathbf{e}_1$ is directed, is "straight". When point M moves along this "straight line", then vectors $\mathbf{e}_2$ and $\mathbf{e}_3$ rotate in the plane which is perpendicular to vector $\mathbf{e}_1$.
Using relation $ds/dt = v$, let's rewrite rotational equations (52) in the form

$$\frac{d\mathbf{e}_1}{dt} = 0 \ , \quad \frac{d\mathbf{e}_2}{dt} = \omega\mathbf{e}_3 \ , \quad \frac{d\mathbf{e}_3}{dt} = -\omega\mathbf{e}_2 \ , \tag{54}$$

where

$$\omega = v\chi \tag{55}$$

- the angular velocity of orientable material point. Since in our case unit vector of particle momentum is defined as

$$\mathbf{p} = m\frac{d\mathbf{x}}{dt},$$

then $\omega = v\chi$ it is possible to interpret as spirality of the particle. If the polarized wave of light propagate along a curve with torsion, then its plane of polarization rotates with an angular velocity (55) (fig.2). Thus torsion generates *own angular rotation of material objects.*

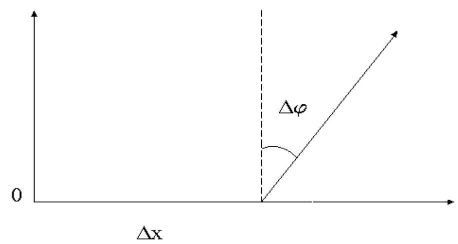

Figure 2: Turn of light beam polarization plane when it is moving along trajectory with $\chi \neq 0$ and $\kappa = 0$

7 Dynamic interpretation of Frenet's equations

From physical point of view the orientable material point represents three-dimensional accelerated reference system, generally having six degrees of freedom - three translational and three rotational. Since Frenet's equations (1-6) describe orientable point motion so that the derivatives of reference vectors are decomposed into that very vectors, then it means, that *Frenet's equations are written down in accelerated reference system.*
It is known from mechanics, that in accelerated reference system equations of motion of *material point* with mass m under action of inertia forces only has the following form [8]

$$\mathbf{F}^{iner} = -m(\mathbf{W} + [\dot{\boldsymbol{\omega}}\mathbf{r}] + [\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]] + 2[\boldsymbol{\omega}\mathbf{v}]) \ . \tag{56}$$

Here

$$\mathbf{F_1} = -m\mathbf{W}$$

- translational force of inertia,

$$\mathbf{F_2} = -m[\dot{\boldsymbol{\omega}}\mathbf{r}]$$

- inertial force connected with rotational acceleration,

$$\mathbf{F_3} = -m[\boldsymbol{\omega}[\boldsymbol{\omega}\mathbf{r}]]$$

- centrifugal force of inertia,

$$\mathbf{F_4} = -2m[\boldsymbol{\omega}\mathbf{v}]$$

- Coriolis force.
In the accompanying reference system ($\mathbf{r} = 0$) equations (56) have the form

$$\mathbf{F}^{iner} = -m\mathbf{W} - 2m[\boldsymbol{\omega}\mathbf{v}]) \ . \tag{57}$$

If besides this three-dimensional rotation of reference system is absent ($\omega = v\chi = 0$), then we have

$$\mathbf{F}^{iner} = -m\mathbf{W}. \tag{58}$$

Comparing equations (51) with (58), we have

$$-\mathbf{W} = \mathbf{e}_2 \kappa v^2 + \mathbf{e}_1 \frac{dv}{dt}. \tag{59}$$

When $dv/dt = 0$ one can see, that in Frenet's equations the curvature of the curve defines *the field of inertia* generating translational force of inertia.

8 Four dimensional Frenet's equations in Riemannian space

Let we have an arbitrary curve in four-dimensional Riemannian space with translational coordinates x^i (i=0,1,2,3). Then the curve is defined by three scalar invariants $\chi_1,\ \chi_2$ *and* χ_3 with the help of four-dimensional Frenet's equations in the following form [9]

$$\frac{D\overset{(0)}{e}_k}{ds} = \chi_1 \overset{(1)}{e}_k, \tag{60}$$

$$\frac{D\overset{(1)}{e}_k}{ds} = \pm\chi_1 \overset{(0)}{e}_k + \chi_2 \overset{(2)}{e}_k, \tag{61}$$

$$\frac{D\overset{(2)}{e}_k}{ds} = \pm\chi_2 \overset{(1)}{e}_k + \chi_3 \overset{(3)}{e}_k, \tag{62}$$

$$\frac{D\overset{(3)}{e}_k}{ds} = \pm\chi_3 \overset{(2)}{e}_k. \tag{63}$$

Here vectors $\overset{(0)}{e}_k, \overset{(1)}{e}_k, \overset{(2)}{e}_k$ *and* $\overset{(3)}{e}_k$ form a tetrad, and through D *the absolute differential* with respect to the four-dimensional Christoffel symbols

$$\Gamma^i{}_{jk} = \frac{1}{2} g^{im}(g_{jm},k + g_{km},j - g_{jk},m) \tag{64}$$

is defined. The signs $\pm$ in these equations are chosen depending on selection of right-hand or left-hand tetrad $\overset{(a)}{e}_k$ (a=0,1,2,3), and also depending on that time-like or space-like is this or that tetrad vector [9].

Statement 3. Any curve of Riemannian space can be considered as the first kind geodesics (the shortest) of space of absolute parallelism, with equations of the form

$$\frac{d^2x^i}{ds^2} = -\Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} - T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}. \tag{65}$$

Proof. Connection of absolute parallelism is defined as [3]

$$\Delta^i{}_{jk} = \Gamma^i{}_{jk} + T^i{}_{jk} = e^i_a e^a{}_{j,k} = -e^a{}_j e^i{}_{a,k}. \tag{66}$$

These relations can be rewritten as follows

$$T^i{}_{jk} = e^i{}_a \nabla_k e^a{}_j = -e^a{}_j \nabla_k e^i{}_a, \tag{67}$$

where ∇_k - covariant derivative with respect to Christoffel symbols (64). Multiplying equality (67) on $e^a{}_i$ $(e^j{}_a)$ and using the orthogonality conditions

$$e^a{}_i e^j{}_a = \delta^j_i, \quad e^a{}_i e^i{}_b = \delta^a_b, \tag{68}$$

where $\delta_i{}^j$ – Cronekker symbols, let's present (67) as follows

$$a)\ \ \nabla_k e^a{}_j = T^a{}_{bk} e^b{}_j \quad or \quad b)\ \ \nabla_k e^i{}_a = -T^i{}_{jk} e^j{}_a. \tag{69}$$

Multiplying (69a) and (69b) on dx^k/ds, we shall obtain

$$\frac{De^a{}_j}{ds} = T^a{}_{bk} e^b{}_j \frac{dx^k}{ds}. \tag{70}$$

$$\frac{De^i{}_a}{ds} = -T^i{}_{jk} e^j{}_a \frac{dx^k}{ds}. \tag{71}$$

Uncovering in equations (71) the absolute differential and supposing in them $e^i{}_{(0)} = dx^i/ds$, we shall obtain geodesics equations (65).
Changing in equations (70) indices on which there is a contraction, we find

$$\frac{De^a{}_k}{ds} = T^a{}_{bj} e^b{}_k \frac{dx^j}{ds}.$$

Choosing in these equations the Frenet's tetrad and writing down them component by component, we have

$$\frac{De^{(0)}_k}{ds} = T^{(0)}_{(1)j} e^{(1)}_k \frac{dx^j}{ds}, \tag{72}$$

$$\frac{De^{(1)}_k}{ds} = T^{(1)}_{(0)j} e^{(0)}_k \frac{dx^j}{ds} + T^{(1)}_{(2)j} e^{(2)}_k \frac{dx^j}{ds}, \tag{73}$$

$$\frac{De^{(2)}_k}{ds} = T^{(2)}_{(1)j} e^{(1)}_k \frac{dx^j}{ds} + T^{(2)}_{(3)j} e^{(3)}_k \frac{dx^j}{ds}, \tag{74}$$

$$\frac{De^{(3)}_k}{ds} = T^{(3)}_{(2)j} e^{(2)}_k \frac{dx^j}{ds}. \tag{75}$$

Comparing equations (60)-(63) with equations (72)-(75), we shall obtain

$$\chi_1 = T^{(0)}_{(1)j} \frac{dx^j}{ds}, \quad \chi_2 = T^{(1)}_{(2)j} \frac{dx^j}{ds}, \quad \chi_3 = T^{(2)}_{(3)j} \frac{dx^j}{ds}.$$

Since the quantities $T^i{}_{kj}$ are defined through Ricci torsion (see (67)), then, as it follows from relations obtained above, *it is possible to geometrize any curves of Riemannian space, using Ricci torsion.*

9 Connection between Ricci rotation coefficients and inertia field in vacuum theory of gravitation

Following Clifford - Einstein program of geometrization of physical equations, the author has found equations of vacuum [3]

$$\nabla_{[k} e^a{}_{m]} - e^b{}_{[k} T^a{}_{|b|m]} = 0, \qquad (A)$$

$$R^a{}_{bkm} + 2\nabla_{[k} T^a{}_{|b|m]} + 2T^a{}_{c[k} T^c{}_{|b|m]} = 0, \qquad (B)$$

which can be represented as an extended set of Einstein-Yang-Mills equations

$$\nabla_{[k} e^a{}_{j]} + T^i{}_{[kj]} e^a{}_i = 0, \qquad (A)$$

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{jm}, \qquad (B.1)$$

$$C^i{}_{jkm} + 2\nabla_{[k} T^i{}_{,|j|m]} + 2T^i{}_{s[k} T^s{}_{,|j|m]} = -\nu J^i{}_{jkm}, \qquad (B.2)$$

with geometrized sources:

$$T_{jm} = -\frac{2}{\nu}\{(\nabla_{[i} T^i{}_{|j|m]} + T^i{}_{s[i} T^s{}_{|j|m]}) - \\ -\frac{1}{2} g_{jm} g^{pn} (\nabla_{[i} T^i{}_{|p|n]} + T^i{}_{s[i} T^s{}_{|p|n]})\} \qquad (76)$$

$$J_{ijkm} = 2g_{[k(i} T_{j)m]} - \frac{1}{3} T g_{i[m} g_{k]j}. \qquad (77)$$

Equations (A) and (B) generalize Einstein vacuum equations

$$R_{ik} = 0 \qquad (78)$$

and solve the problem of geometrization of energy-momentum tensor, proposed by A.Einstein [3]. Completely geometrized equations of gravitational field $(B.1)$ contain in its right hand side the energy-momentum tensor (76), formed by Ricci rotation coefficients and their derivatives, i.e. Ricci torsion.

Gravitational field theory, based on vacuum equations (A) and (B), allows to establish the connection between Ricci rotation coefficients and inertia fields and forces.

In order to prove this let's write down the vacuum equations in spinor basis with the help of Newman-Penrose spinor coefficients [10] and Carmeli spinor matrixes [11]

$$\partial_{C\dot{D}} \sigma^i{}_{A\dot{B}} - \partial_{A\dot{B}} \sigma^i{}_{C\dot{D}} = (T_{C\dot{D}})_A{}^P \sigma^i{}_{P\dot{B}} + \sigma^i{}_{A\dot{R}} (T^+{}_{\dot{D}C})^{\dot{R}}{}_{\dot{B}} - \\ -(T_{A\dot{B}})_C{}^P \sigma^i{}_{P\dot{D}} - \sigma^i{}_{C\dot{R}} (T^+{}_{\dot{B}A})^{\dot{R}}{}_{\dot{D}} \, , \qquad (\overset{+}{A}{}^s)$$

$$2\Phi_{AB\dot{C}\dot{D}} + \Lambda \varepsilon_{AB} \varepsilon_{\dot{C}\dot{D}} = \nu T_{A\dot{C}B\dot{D}} \, , \qquad (\overset{+}{B}{}^{s+}.1)$$

$$C_{A\dot{B}C\dot{D}} - \partial_{C\dot{D}} T_{A\dot{B}} + \partial_{A\dot{B}} T_{C\dot{D}} + (T_{C\dot{D}})_A{}^F T_{F\dot{B}} + (T^+{}_{\dot{D}C})^{\dot{F}}{}_{\dot{B}} T_{A\dot{F}} - \\ -(T_{A\dot{B}})_C{}^F T_{F\dot{D}} - (T^+{}_{\dot{B}A})^{\dot{F}}{}_{\dot{D}} T_{C\dot{F}} - [T_{A\dot{B}} \, , T_{C\dot{D}}] = -\nu J_{A\dot{B}C\dot{D}} \, , \qquad (\overset{+}{B}{}^{s+}.2)$$

$$A, C... = 0, 1 \, , \qquad \dot{B}, \dot{D}... = \dot{0}, \dot{1} \, .$$

There is the solution of these equations leading to Schwarzschild-type metric

$$ds^2 = \left(1 - \frac{2\Psi^0}{r}\right) c^2dt^2 - \left(1 - \frac{2\Psi^0}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{79}$$

where

$$\Psi^0 = MG/c^2, \tag{80}$$

in combining form will be rewritten as [3]:
1. Coordinates: $u,\ r,\ x^2\ and\ x^3$.
2. Components of Newman-Penrose coefficients:

$$\sigma^i_{0\dot{0}} = (0,1,0,0), \quad \sigma^i_{1\dot{1}} = (1,U,0,0), \quad \sigma^i_{0\dot{1}} = \rho(0,0,P,iP),$$

$$\sigma_i^{0\dot{0}} = (1,0,0,0), \quad \sigma_i^{1\dot{1}} = (-U,1,0,0), \quad \sigma_i^{0\dot{1}} = -\frac{1}{2\rho P}(0,0,1,i),$$

$$U = -1/2 + \Psi^0/r, \quad P = (2)^{-1/2}(1+\zeta\overline{\zeta}/4), \quad \zeta = x^2 + ix^3,$$

$$\Psi^0 = \text{const}.$$

3. Spinor components of Ricci rotation coefficients:

$$\rho = -1/r, \quad \alpha = -\overline{\beta} = -\alpha^0/r, \quad \gamma = \Psi^0/2r,$$

$$\mu = -\varepsilon^0/r + 2\Psi^0/r^2, \alpha = \zeta/4.$$

4. Spinor components of the Riemann tensor:

$$\Psi = -\Psi^0/r^3.$$

Using this solution, it is possible to calculate gravitational fields ($\Gamma^i{}_{jk}$) and Ricci rotation coefficients ($T^i{}_{jk}$) in geodesics equations (65), which in vacuum theory of gravitation are regarded as equations of motion of orientable material point.
In theory of physical vacuum equations of motion of prob particle coincide with geodesics equations of absolute parallelism space (65).
For simplicity we shall pass in given solution to quasicartesian coordinates, in which Schwarzschild-type metric has the form

$$ds^2 = (1 - \frac{2MG}{rc^2})c^2dt^2 - (1 + \frac{2MG}{rc^2})(dx^2 + dy^2 + dz^2)\ . \tag{81}$$

In these coordinates the tetrad $e^a{}_i$ takes the following form

$$e^{(0)}{}_0 = (1 + \frac{2\varphi}{c^2})^{1/2}\ , \tag{82}$$

$$e^{(1)}{}_1 = e^{(2)}{}_2 = e^{(3)}{}_3 = (1 - \frac{2\varphi}{c^2})^{1/2}\ ,$$

where in brackets tetrad indices are designated and $\varphi = -MG/r$.

Metric tensor for tetrad (82) can be obtained with the help of relations

$$g_{ik} = \eta_{ab} e^{a}{}_{i} e^{b}{}_{k} \ , \quad \eta_{ab} = \eta^{ab} = \mathrm{diag}(1 - 1 - 1 - 1). \tag{83}$$

Considering non-relativistic approximation and supposing fields to be weak, i.e. supposing that

$$\frac{2\varphi}{c^2} << 1 \ , \quad g_{ik} \simeq \eta_{ik} \ , \quad ds \simeq ds_0 = cdt(1 - \frac{v^2}{c^2})^{1/2} \ , \tag{84}$$

$$R^{i}{}_{jkm} \simeq \overset{o}{R}{}^{i}{}_{jkm} = 0 \ , \quad \frac{v^2}{c^2} << 1 \ , \quad ds \simeq ds_0 \simeq cdt \ ,$$

we find from equations (65) the following approximate equations of motion of mass m

$$m\frac{d^2x^\alpha}{dt^2} = -mc^2(\Gamma^{\alpha}{}_{00} + T^{\alpha}{}_{00}), \tag{85}$$

$$\alpha = 1, 2, 3.$$

Using metric (81) and tetrad (82) and also formulas (64) and (67), we find

$$\Gamma^{\alpha}{}_{00} = -MGx^\alpha/r^3, \quad T^{\alpha}{}_{00} = MGx^\alpha/r^3.$$

Comparing equations (85) with classical mechanics equations in attendant reference system [8]

$$m\frac{d^2x^\alpha}{dt^2} = F^{\alpha}_{G} - mW^\alpha = 0 \ , \tag{86}$$

we get that

$$F^{\alpha}_{G} = -mc^2\Gamma^{\alpha}{}_{00} = mMGx^\alpha/r^3 \tag{87}$$

represents gravitational force generated by gravitational field

$$\Gamma^{\alpha}{}_{00} = -MGx^\alpha/r^3,$$

and

$$-mW^\alpha = -mc^2 T^{\alpha}{}_{00} = -mMGx^\alpha/r^3 \tag{88}$$

is inertial force generated by inertial field

$$T^{\alpha}{}_{00} = MGx^\alpha/r^3.$$

These forces compensate each other, locally creating condition of weightlessness.
Thus by direct calculations on particular example it was shown that Ricci rotation coefficients describe inertial fields generating inertial forces. Therefore nature of inertial fields and forces is connected with Ricci torsion of space - time, with consistent description of inertial fields and forces requiring introduction of the geometry of absolute parallelism [3].

10 Ricci torsion in vacuum electrodynamics

From equations of vacuum (A) and (B) the equations of vacuum electrodynamics follow [3]. In this electrodynamics the effects of Ricci torsion generated by spin of charged particles are exhibited.

As in usual electrodynamics in vacuum electrodynamics approximate equations of motion of radiating charge are written as

$$m\ddot{\mathbf{x}} = e\mathbf{E} + \frac{e}{c}[\dot{\mathbf{x}}\mathbf{H}] + \frac{2e^2}{3c^3}\,\dddot{\mathbf{x}}\ , \tag{89}$$

however in vacuum electrodynamics the reaction force of the radiation

$$\frac{2e^2}{3c^3}\,\dddot{\mathbf{x}}$$

contains Ricci torsion generated by spin of the charge. Indeed, choosing time t as a parameter in equations (1-3) one can see, that

$$\dot{\mathbf{x}} = v\mathbf{e}_1, \quad \ddot{\mathbf{x}} = a\mathbf{e}_1 + kv^2\mathbf{e}_2 \quad \dddot{\mathbf{x}}= (\dot{a} - \kappa^2 v^3)\mathbf{e}_1 + (3va\kappa + v^2\dot{\kappa})\mathbf{e}_2 + \kappa\chi v^3\mathbf{e}_3,$$

where

$$\dot{\mathbf{x}} = d\mathbf{x}/dt, \quad ds = vdt, \quad a = \dot{v}, \quad \dot{a} = d^3/dt^3 \quad \dot{\kappa} = d\kappa/dt.$$

For reaction force of the radiation we have

$$\mathbf{F}_{rad} = \frac{2e^2}{3c^3}\{(\dot{a} - \kappa^2 v^3)\mathbf{e}_1 + (3va\kappa + v^2\dot{\kappa})\mathbf{e}_2 + \kappa\chi v^3\mathbf{e}_3\}. \tag{90}$$

From these equations one can see that the reaction force of the radiation in vacuum electrodynamics has complex structure. It contains terms generated not only by external electromagnetic fields, but also by torsion. The last term in right hand side of equation (90) contain torsion χ, therefore *accelerated particle possessing a spin, radiates at the same time electromagnetic and torsion fields (fields of Ricci torsion).* This theoretical conclusion is excellently confirmed by numerous experimental facts [7].

It is necessary to note that until now special experiments on research of structure of the reaction force of the radiation were not carried out. Only the surprising N.Tesla devices are known permitting to transmit electromagnetic energy by a way, not explained by conventional electrodynamics.

10.1 Theoretical evaluation of electrotorsion radiation in vacuum electrodynamics

Using relation (90), it is possible to produce approximate evaluation of magnitude of force of electrotorsion interaction and to compare it with forces of electromagnetic and gravitational interactions. For this purpose we shall consider an electron as a sphere having radius equal to Compton radius of an electron

$$\lambda = \frac{\hbar}{mc} = 3,6 \times 10^{-11} sm. \tag{91}$$

All calculations we shall conduct in SGSE system. Let us present spin of electron as

$$s = J\omega = Jv\chi = \frac{\hbar}{2}, \tag{92}$$

where the moment of inertia J of electron is calculated as a moment of inertia of the sphere with radius (91)

$$J = \frac{2}{5} m \lambda^2,$$

and $\omega = v\chi$ - the angular velocity of rotation of electron. From relation (92) we find this quantity for electron

$$\omega \approx 10^{21} rad/s. \tag{93}$$

Let us suppose now that the electron radiates at transition from one stationary level to another in atom of hydrogen. Let thus it approximately be the first Bohr orbit ($E \approx 10^8\ V/sm$). Then it is easy to calculate the force of electromagnetic $\mathbf{F}_e$ and gravitational $\mathbf{F}_g$ interaction of electron with the nucleus:

$$|\mathbf{F}_e| = eE = m\kappa_e^{iht} = mv^2\kappa_e = \frac{e^2}{r_0^2} \approx 4,8 \times 10^{-2} \text{ din}, \tag{94}$$

$$|\mathbf{F}_g| = mG = m\kappa_g^{int} = mv^2\kappa_g = \frac{\gamma m M_n}{r_0^2} \approx 0,6 \times 10^{-42}\ din.$$

From the equality (90) for the force of electrotorsion interaction we find

$$|\mathbf{F}_{\kappa\chi}| = \frac{2e^2}{3c^3} \kappa^{int} \omega. \tag{95}$$

With the help of formula (94) we shall obtain $\kappa^{int} = v^2\kappa \approx 10^{25}\ sm/s^2$.

Substituting this quantity in (95) and taking into account (93), we find the value of electrotorsion interaction force

$$F_{\kappa\chi} \approx 2,9 \times 10^{-4}\ din. \tag{96}$$

Thus, *the electrotorsion force of electron radiation in the nucleus appears to be weaker than electrostatic force and stronger than force of gravitational interaction*, that also is observed in experiment [7].

11 Theoretical research of physical properties of torsion fields

This section includes a broad circle of problems, therefore, for brevity, we shall restrict ourselves only by enumeration of properties of torsion fields, basing on equations of vacuum (A) and (B).

As it was shown in [3], in the theory of vacuum there are two types of torsion fields generated by Ricci torsion:

a) primary torsion fields generated by Absolute "Nothing";

b) secondary torsion fields generated by matter.

11.1 Properties of primary torsion fields

Primary torsion fields are the space-time vortexes satisfying equations [3]

$$\nabla_{[i} T^i{}_{|j|m]} + T^i{}_{s[i} T^s{}_{|j|m]} = 0, \tag{97}$$

Comparing this relation with definition of energy-momentum tensor of matter (76) in equations of vacuum (A) and (B), we get zero value of this tensor for primary torsion field

$$E = \int T^{jm} g_{jm} (-g)^{\frac{1}{2}} \, dV \equiv 0. \tag{98}$$

The exact solution of vacuum equations in this case shows that torsion field is distinct from zero and capable to rotate a plane of polarization of polarized light wave [3]. Here we have the case, when the field $T^i{}_{jk}$ bears the information without energy transmission. The trajectory of a probe particle in primary torsion field will vary under operation of a field according to equations of motion

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} + T^i{}_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0,$$

but in this interaction the energy of particle remains constant (interaction without energy).

For an object, which energy is equal to zero, it is impossible to formulate a concept of speed of its propagation. For the usual observer such object is "at once everywhere and always", i.e. its "speed of propagation" is instantaneous.

The creation of primary torsion fields can be considered as primary polarization of vacuum according to its spin properties, with right-hand and left-hand fields simultaneously arising .

The experiments on creation of artificial torsion polarization of vacuum by introducing in some its area a material objects with various surface geometry show [7], that right-hand and left-hand primary torsion fields simultaneously arise. The geometry of space in this case represents 10-dimensional manifold (4 translational coordinates and 6 angular ones), and its Riemannian curvature $R^i{}_{jkm}$ being equal to zero, and Ricci torsion being distinct from zero and satisfying to equations (97).

"Propagation" of primary torsion fields with "instantaneous speeds" happens on phase portrait of these fields, but not with the help of group velocity, as it happens with usual physical fields. It indicates a holographic structure of torsion fields.

11.2 Properties of secondary torsion fields

Secondary torsion fields are connected with rotation of material objects. They substantially save the properties of primary torsion fields, however, in difference from the last, in a bound state they can considerably change potential energy of material systems. For example, the potential of interaction of spinning mass M, found on the base of precise solution of the vacuum equation, looks like

$$\varphi_s = -\frac{MGr}{r^2 + r^2{}_s cos^2\theta}, \tag{99}$$

where r_s - Kerr parameter [3]. When the rotation of the mass is absent, this parameter becomes equal to zero and we have usual Newton potential. On distances $r \approx r_s$ the rotation gives significant contribution to potential energy of interaction.

It is possible to show that parameter $r \approx r_s$ generates torsion

$$\chi(s) = T^{(2)}{}_{(3)\gamma} \frac{dx^\gamma}{ds} \tag{100}$$

in nonrelativistic equations of motion and results in energy changes during motion of a system. Let now in ratio (99) $M = 0$, but $r_s \neq 0$, that corresponds to the solution for primary torsion field. It is easy to see that the potential (99) in this case becomes equal to zero, and torsion field (100) is distinct from zero and is capable to transmit information. This result can be treated as a capability of secondary torsion fields to become free, gaining the properties of primary ones. In the given example vanishing of the potential of interaction indicates high penetrating ability of secondary (and primary) torsion fields, if they are not in a bound state.

Collecting outcomes, let us enumerate the main properties of torsion fields obtained as a result of the theoretical analysis of the vacuum equations:

- Information transmission without carrying energy.
- The speed of propagation is infinite.
- High penetrating ability.
- Holographic nature.
- Ability in a bound state to change energy.

References

[1] *Frenet F.* Jour. de Math. 1852. Vol. 17. P. 437-447.

[2] *Ricci G.* Mem.Acc.Linc. 1895. Vol. 2. Ser. 5. P. 276-322.

[3] *Shipov G.I.* A Theory of a physical vacuum, M.: Nauka, 1997. 450 p.

[4] *Schouten J.A. , Stroyk D.D.* Introduction into new methods of differential geometry. V.2. M.: GIIL, 1948.

[5] *Cartan E., Schouten J. //* Proc. Knkl. nederl. akad. 1926. Vol. 29. P. 803-810.

[6] *Cartan E., Schouten J. //* Ibid. P. 933-938.

[7] *Akimov A.E.* Heuristic discussion of search for new long-range actions. The EGS-concepts. M.: CISE VENT, 1991, 63 p.

[8] *Ol'hovskiy I.I.* Course of theoretical mechanics for physicist. M.: Nauka, 1970.

[9] *Rashevskiy P.K.* Riemannian geometry and tensor analysis. M.: Nauka, 1964.

[10] *Newman E., Penrose R. //* J. Math. Phys. 1962. Vol. 3, No 3. P.566-587.

[11] *Carmeli M.//* Phys. Rev.D. 1972. Vol. 5. P. 5-8.