Dark Energy in the Theory of Physical Vacuum

G. Shipov
shipov@aha.ru, website http://www.shipov.com

Introduction

Einstein believed that one of the main problems in unified field theory is the one of the geometrization of the energy-momentum tensor of matter on the right-hand side of his equations. This problem can be solved using the geometry of absolute parallelism and Cartan's structural equations in this geometry [1]:

$$\nabla_{[k} e^a{}_{m]} - e^b{}_{[k} T^a{}_{|b|m]} = 0, \tag{A}$$

$$R^a{}_{bkm} + 2\nabla_{[k} T^a{}_{|b|m]} + 2T^a{}_{c[k} T^c{}_{|b|m]} = 0, \tag{B}$$

$$i, j, k... = 0, 1, 2, 3, \qquad a, b, c... = 0, 1, 2, 3.$$

These equations are essentially a matrix form, where the matrices $e^a{}_m$, $T^a{}_{bm}$ and $R^a{}_{bkm}$ appear as the main gauge potential and fields in the theory of physical vacuum.

1 Vacuum equations as an extended set of the Einstein-Yang-Mills equations. Geometrization of matter fields

The geometry of absolute parallelism is a space of events in the theory of physical vacuum. Speaking here of events, we mean the interaction of whole and part in a certain physical situation.

Consider an event that represents some excitation of physical vacuum and show that this excitation is described by Einstein-like equations with geometrized energy-momentum tensor.

Contracting the equations (B) written as

$$R^i{}_{jkm} + 2\nabla_{[k} T^i{}_{|j|m]} + 2T^i{}_{s[k} T^s{}_{|j|m]} = 0 \tag{1}$$

in indices i and k, we will have

$$R_{jm} = -2\nabla_{[i} T^i{}_{|j|m]} - 2T^i{}_{s[i} T^s{}_{|j|m]}. \tag{2}$$

Contracting further the equations (2) by the metric tensor g^{jm}, we get

$$R = -2g^{jm}(\nabla_{[i} T^i{}_{|j|m]} + T^i{}_{s[i} T^s{}_{|j|m]}). \tag{3}$$

Forming, using (2) and (3), the Einstein tensor

$$G_{jm} = R_{jm} - \frac{1}{2} g_{jm} R,$$

we will get the equations

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{jm}, \tag{4}$$

similar to Einstein's equations, but with the geometrized right-hand side, defined as

$$T_{jm} = -\frac{2}{\nu}\{(\nabla_{[i} T^{i}{}_{|j|m]} + T^{i}{}_{s[i} T^{s}{}_{|j|m]}) -$$
$$-\frac{1}{2} g_{jm} g^{pn} (\nabla_{[i} T^{i}{}_{|p|n]} + T^{i}{}_{s[i} T^{s}{}_{|p|n]})\} \tag{5}$$

Using the notation

$$P_{jm} = (\nabla_{[i} T^{i}_{|j|m]} + T^{i}{}_{s[i} T^{s}_{|j|m]})$$

we will have from (5)

$$T_{jm} = -\frac{2}{\nu}(P_{jm} - \frac{1}{2} g_{jm} g^{pn} P_{pn}). \tag{6}$$

Tensor (6) can be represented as the sum of parts symmetrical and skew-symmetrical in the indices j and m, and

$$R_{jm} - \frac{1}{2} g_{jm} R = \nu T_{(jm)}, \tag{7}$$

$$T_{[jm]} = \frac{1}{\nu}(-\nabla_i \Omega_{jm}^{\cdot\cdot i} - \nabla_m A_j - A_s \Omega_{jm}^{\cdot\cdot s}) = 0, \tag{8}$$

$$A_j = T^{i}{}_{ji}.$$

Relationship (8) can be taken to be the equations obeyed by the torsion $\Omega_{jm}^{\cdot\cdot i}$ of absolute parallelism geometry [1], which form the energy-momentum tensor (5).

[1] In the article " Riemann-Geometrie mit Aufrechterhaltung des Bergiffes des Fernaparralelismus" Sitzungsber. preuss. Akad.Wiss., phys.-math. Kl., 1928, 217-221 A.Einstein used the torsion of absolute parallelism in the formula (10) determining torsion as

$$\Lambda^{\alpha}{}_{\beta\gamma} = \frac{1}{2}(\Delta^{\alpha}{}_{\beta\gamma} - \Delta^{\alpha}{}_{\gamma\beta}),$$

where

$$\Delta^{\alpha}{}_{\beta\gamma} = e^{\alpha}_{a} e^{a}_{\beta}, \gamma, \quad , \gamma = \frac{\partial}{\partial x_{\gamma}}, \quad \alpha, \beta, ... = 0, 1, 2, 3, \quad a, b... = 0, 1, 2, 3$$

–connection of absolute parallelism,

$$\Lambda^{\alpha}{}_{\beta\gamma} = -\Omega_{jm}^{\cdot\cdot i}$$

– anholonomity object in J. Schouten definition. In the same article A. Einstein has specified, that when torsion $\Lambda^{\alpha}{}_{\beta\gamma}$ (anholonomity object) is equal to zero the space becomes Minkovski space. Object of anholonomity in the theory appears when we use the manifold of oriented points and anholonomic tetrad. Tetrad formulation of Einstein equations connects curvature $R^{i}{}_{jkm}$ with Ricci rotation coefficients $T^{i}{}_{jk}$ us

$$R^{i}{}_{jkm} = -2\nabla_{[k} T^{i}_{|j|m]} - 2 T^{i}_{c[k} T^{c}_{|j|m]},$$

where

$$T^{i}_{jk} = -\Omega_{jk}^{\cdot\cdot i} + g^{im}(g_{js}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s})$$

We now decompose the Riemann tensor R_{ijkm} into the irreducible parts

$$R_{ijkm} = C_{ijkm} + g_{i[k}R_{m]j} + g_{j[k}R_{m]i} + \frac{1}{3}Rg_{i[m}g_{k]j}. \tag{9}$$

Using equations (4), written as

$$R_{jm} = \nu(T_{jm} - \frac{1}{2}g_{jm}T), \tag{10}$$

we rewrite (9) as

$$R_{ijkm} = C_{ijkm} + 2\nu g_{[k(i}T_{j)m]} - \frac{1}{3}\nu T g_{i[m}g_{k]j}, \tag{11}$$

where T is the trace of (6).

Let us now introduce the tensor current

$$J_{ijkm} = 2g_{[k(i}T_{j)m]} - \frac{1}{3}T g_{i[m}g_{k]j} \tag{12}$$

and represent the tensor (11) as the sum

$$R_{ijkm} = C_{ijkm} + \nu J_{ijkm}. \tag{13}$$

Substituting this relationship into (1), we will obtain

$$C_{ijkm} + 2\nabla_{[k}T_{|ij|m]} + 2T_{is[k}T^{s}{}_{|j|m]} = -\nu J_{ijkm}. \tag{14}$$

Equations (14) represent the Yang-Mills equations with a geometrized source given by (3.12). In (3.14) the role of Yang-Mills field is played by the Weyl tensor C_{ijkm}, and that of potentials by the Ricci rotation coefficients $T^{i}{}_{jk}$.

Let us now substitute (13) into the second Bianchi identities in the geometry of absolute parallelism [20]

$$\nabla_{[n}R_{\ |ij|km]} + R^{s}{}_{j[km}T_{\ |is|n]} - T^{s}{}_{j[n}R_{\ |is|km]} = 0. \tag{15}$$

As a result we have the equations of motion

$$\nabla_{[n}C_{\ |ij|km]} + C^{s}{}_{j[km}T_{\ |is|n]} - T^{s}{}_{j[n}C_{\ |is|km]} = -\nu J_{nijkm} \tag{16}$$

for the Yang-Mills field C_{ijkm}. The source J_{nijkm} in these equations will then by defined through the current (3.12) as follows:

$$J_{nijkm} = \nabla_{[n}J_{\ |ij|km]} + J^{s}{}_{j[km}T_{\ |is|n]} - T^{s}{}_{j[n}J_{\ |is|km]}. \tag{17}$$

Using the geometrized Einstein equations (4) and the Yang-Mills equations (14), we can represent the equations of physical vacuum (A) and (B) in the form of the extended set of the Einstein-Yang-Mills equations

$$(A) \qquad \nabla_{[k}e^{a}{}_{j]} + T^{i}{}_{[kj]}e^{a}{}_{i} = 0,$$

–Ricci rotation coefficients (torsion field). If in tetrad formulation to put object of anholonomity equal to zero Riemann curvature addresses in zero. From my point of view anholonomity has new physical sense - we enter as elements of space-time new rotational degrees of freedom - anholonomic rotational coordinates.

$(B.1)$ $$R_{jm} - \frac{1}{2}g_{jm}R = \nu T_{jm} \ ,$$

$(B.2)$ $$C^i{}_{jkm} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i{}_{s[k}T^s_{|j|m]} = -\nu J^i{}_{jkm},$$

where the geometrized sources T_{jm} and J_{ijkm} are given by (5) and (12).

For the case of Einstein's vacuum the equations (A) and (B) are significantly simplified and become

(I) $$\nabla_{[k}e^a{}_{j]} + T^i{}_{[kj]}e^a{}_i = 0,$$

(II) $$R_{jm} = 0,$$

(III) $$C^i{}_{jkm} + 2\nabla_{[k}T^i_{|j|m]} + 2T^i{}_{s[k}T^s_{|j|m]} = 0.$$

2 Inertial field as a torsion field

The equations of motion for an oriented point in the theory of physical vacuum coincide with the equations of the geodesics of the space of absolute parallelism

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} + T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0, \tag{18}$$

which differ from the equations of motion in Einstein's theory of gravitation by the additional term

$$T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}.$$

The very name of the quantities

$$T^i{}_{jk} = e^i{}_a\nabla_k e^a{}_j \tag{19}$$

–the Ricci rotation coefficients suggests that they describe rotation. It follows from (19) that the quantities $T^i{}_{jk}$ describe the change in the orientation of the tetrad vectors $e^a{}_j$ when the tetrad shifts by an infinitesimal distance dx^i (the covariant derivative ∇_k is taken with respect to the connection $\Gamma^i{}_{jk}$, therefore in "normal" coordinates $\nabla_k = \partial_k$). Using the Ricci rotation coefficients we can form the four-dimensional angular velocity of rotation of the tetrad vector

$$\Omega^i{}_j = T^i{}_{jk}\frac{dx^k}{ds} \tag{20}$$

with the symmetry properties

$$\Omega_{ij} = -\Omega_{ji}. \tag{21}$$

Suppose now that the tetrad vectors coincide with the vectors of a four-dimensional arbitrarily accelerated reference frame, then, by (20), the rotation of the reference frame is fully determined by the torsion field $T^i{}_{jk}$. Since the field $T^i{}_{jk}$ transforms following a tensor

law relative to the coordinates transformations x_i, the rotation of reference frames relative to the coordinate transformations is absolute. The nontensor transformation law of $T^i{}_{jk}$ is valid for transformations in the angular coordinates $\varphi_1, \varphi_2, \varphi_3, \theta_1,$
θ_2, θ_3, therefore rotation is only relative for the group of rotations $O(3.1)$ [1].

Let us now write the nonrelativisitc equations of motion of a mass m under inertia forces alone, assuming that at a given moment of time it passes through the origin of an accelerated system

$$\frac{d}{dt}(m\mathbf{v}) = m(-\mathbf{W} + 2[\mathbf{v}\omega]). \tag{22}$$

These equations can be written in the form

$$\frac{d}{dt}(mv_\alpha) = m(-W_{\alpha o} + 2\omega_{\alpha\beta}\frac{dx^\beta}{dt}), \quad \alpha, \beta = 1, 2, 3, \tag{23}$$

where $\mathbf{W} = (W_1, W_2, W_3) = (W_{10}, W_{20}, W_{30}), \omega = (\omega_1, \omega_2, \omega_3)$,

$$\omega_{\alpha\beta} = -\omega_{\beta\alpha} = -\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}. \tag{24}$$

On the other hand, equations (18), if we take into account (20), can be represented as

$$\frac{d^2x^i}{ds^2} + \Omega^i{}_j\frac{dx^j}{ds} = 0. \tag{25}$$

Multiplying these equations by mass m, we will write the nonrelativistic three-dimensional part of these equations in the form

$$m\frac{du_\alpha}{ds_0} = -m\Omega_{\alpha 0}\frac{dx^0}{ds_0} - 2m\Omega_{\alpha\beta}\frac{dx^\beta}{ds_0}. \tag{26}$$

Since in a nonrelativistic approximation

$$ds_0 = cdt, u_\alpha = \frac{v_\alpha}{c}$$

and $dx_0 = cdt$, then the equations (26) become

$$m\frac{dv_\alpha}{dt} = -mc^2\Omega_{\alpha 0} - 2mc^2\Omega_{\alpha\beta}\frac{1}{c}\frac{dx^\beta}{dt}. \tag{27}$$

Comparing (27) with (23) gives

$$\Omega_{10} = \frac{W_1}{c^2}, \quad \Omega_{20} = \frac{W_2}{c^2},$$
$$\Omega_{30} = \frac{W_3}{c^2}, \quad \Omega_{12} = -\frac{\omega_3}{c},$$
$$\Omega_{13} = \frac{\omega_2}{c}, \quad \Omega_{23} = -\frac{\omega_1}{c}.$$

Consequently, the matrix of the four-dimensional angular velocity of rotation of an arbitrarily accelerated reference frame (matrix of the four-dimensional "classical spin") has the form

$$\Omega_{ij} = \frac{1}{c^2}\begin{pmatrix} O & -W_1 & -W_2 & -W_3 \\ W_1 & 0 & -c\omega_3 & c\omega_2 \\ W_2 & c\omega_3 & 0 & -c\omega_1 \\ W_3 & -c\omega_2 & c\omega_1 & 0 \end{pmatrix}. \tag{28}$$

It is seen from the matrix that the four-dimensional rotation of a reference frame caused by the inertial fields $T^i{}_{jk}$ is associated with the rotation

$$\Omega_{jk}^{\cdot\cdot i} = -T^i{}_{[jk]} \tag{29}$$

of a space of events in universal relativity theory. Fields determined by the rotation of space came to be known as torsion fields. Accordingly, the torsion field $T^i{}_{jk}$ represents the inertial field engendered by the torsion of a space of absolute parallelism.

3 Inertial field in an inertial frame

Most of modern physical theories have been formulated for inertial reference frames. A reference frame is inertial if it moves linearly and uniformly (without rotation) relative to another one, similar to it. In actual fact, we have here two inertial frames defined through each other.

There is another definition of the inertial reference frame – based on the concepts of inertial fields and inertia forces. We then have the following definition: a reference frame is inertial if there is no inertia forces in it.

Since inertia forces are engendered by an inertial field, it seems that inertial fields in inertial reference frame must vanish as well. But for the universal theory of relativity this is not the case – inertial fields (or torsion fields) are nonzero even in inertial reference frames. Indeed, in inertial reference frame the inertia force in (18) becomes zero

$$F_I^i = mT^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \tag{30}$$

In this equation the inertial field $T^i{}_{jk}$ is defined through torsion $\Omega_{jk}^{\cdot\cdot i} = -\Omega_{kj}^{\cdot\cdot i}$ of a space of absolute parallelism

$$\Omega_{jk}^{\cdot\cdot i} = e^i{}_a e^a{}_{[k,j]} = \frac{1}{2}e^i{}_a(e^a{}_{k,j} - e^a{}_{j,k}) \tag{31}$$

in the following manner [1]:

$$T^i{}_{jk} = -\Omega_{jk}^{\cdot\cdot i} + g^{im}(g_{js}\Omega_{mk}^{\cdot\cdot i} + g_{ks}\Omega_{mj}^{\cdot\cdot i}). \tag{32}$$

Substituting this relationship into (30) gives (for $m \neq 0$)

$$-\Omega_{jk}^{\cdot\cdot i}\frac{dx^j}{ds}\frac{dx^k}{ds} + g^{im}(g_{gs}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s})\frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \tag{33}$$

Since the relationship

$$\frac{dx^j}{ds}\frac{dx^k}{ds}$$

is symmetrical in indices j and k, and the rotation $\Omega_{jk}^{\cdot\cdot\ i}$ is skew-symmetrical in these indices, the first term in (33) goes to zero. As a result, it follows from (33) that

$$g^{im}(g_{js}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s}) = 0, \tag{34}$$

or

$$(g_{js}\Omega_{mk}^{\cdot\cdot s} + g_{ks}\Omega_{mj}^{\cdot\cdot s}) = 0, \tag{35}$$

hence

$$\Omega_{mkj} = -\Omega_{mjk}. \tag{36}$$

Since Ω_{mkj} is skew-symmetric in indices m and k, then it follows from (36) that in inertial reference frames the torsion of a space of absolute parallelism is skew-symmetrical in all the three indices.

Substituting (35) into (32) gives

$$T_{ijk} = -T_{jik} = -T_{ikg} = -\Omega_{ijk}. \tag{37}$$

Therefore, in inertial frames the equation (8) becomes simpler in form:

$$\nabla_i\Omega_{jm}^{\cdot\cdot i} = 0, \tag{38}$$

The energy-momentum tensor (5) is symmetrical in the indices j and m to yield

$$T_{jm} = \frac{1}{\nu}(\Omega_{sm}^{\cdot\cdot i}\Omega_{ji}^{\cdot\cdot s} - \frac{1}{2}g_{jm}\Omega_{s}^{\cdot ji}\Omega_{ji}^{\cdot\cdot s}). \tag{39}$$

We can define the auxilliary pseudovector h_m as follows

$$\Omega^{ijk} = \varepsilon^{ijkm}h_m, \quad \Omega_{ijk} = \varepsilon_{ijkm}h^m \tag{40}$$

where ε_{ijkn} is fully skew-symmetrical Levi-Civita symbol and write the tensor (39) as

$$T_{jm} = \frac{1}{2\nu}(h_j h_m - \frac{1}{2}g_{jm}h^i h_i). \tag{41}$$

Substituting (40) into (38) gives

$$h_{m,j} - h_{j,m} = 0. \tag{42}$$

These equations have two solutions: $h_m = 0$ (a trivial one), and

$$h_m = \Psi_{,m}, \tag{43}$$

where Ψ is pseudoscalar.

Writing the energy-momentum tensor (41) through this pseudoscalar, we have

$$T_{jm} = \frac{1}{2\nu}(\Psi_{,j}\Psi_{,m} - \frac{1}{2}g_{jm}\Psi^{,i}\Psi_{,i}). \tag{44}$$

In quantum field theory the tensor (44) is the energy-momentum tensor of a massless pseudoscalar field, where the pseudoscalar Ψ plays the role of the wave function in quantum equations of motion.

If the pseudovector h_m is time-like, it can conveniently be represented as

$$h_m = \Psi_{,m} = \varphi(x^i)u_m, \tag{45}$$

where

$$u_m u^m = 1 \tag{46}$$

and $\varphi(x^i)$ is a scalar quantity.

Substitution of (45) into the tensor (41) yields the energy-momentum tensor of the form

$$T_{jm} = \frac{1}{\nu}\varphi^2(u_j u_m - \frac{1}{2}g_{jm}). \tag{47}$$

The tensor (47) in its structure looks rather like the energy-momentum tensor of an ideal liquid.

Let us show that matter described in inertial reference frames by energy-momentum tensors of the form (47) is relative, i.e., it obeys the principle of universal relativity.

Defining the density of matter as

$$\rho = T/c^2, \tag{48}$$

where

$$T = g^{jm}T_{jm}, \tag{49}$$

we find from (37), (39) and (41)

$$\rho = T/c^2 = -\frac{1}{2\nu c^2}h^j h_j = -\frac{1}{\nu c^2}\Omega_s^{\cdot ji}\Omega_{ji}^{\cdot\cdot\, s} = -\frac{1}{\nu c^2}T_s^{\ ji}T_{ji}^{\ \ s}. \tag{50}$$

Suppose now that the density (50) describes the mass of a vacuum purely field particle with the energy-momentum tensor (39), then the mass can be written as

$$m_0 = \int \rho(-g)^{1/2}dV, \tag{51}$$

where

$$g = \det g_{jm}, \qquad dV = dx^1 dx^2 dx^3,$$

and the density ρ is given by (50).

Density (50), and hence the mass (51), behave as absolute magnitudes relative to coordinate transformations x_i, since the inertial field T_{ijk} is a tensor relative to these transformations. Using the tetrad $e^a_{\ i}$ we can pass over from the base indices $i, j, k...$ to the fiber indices $a, b, c....$ For example,

$$T^a_{\ bk} = e^a_{\ i}T^i_{\ jk}e^j_{\ b}.$$

Using this property and the orthogonality conditions $e^i_{\ a}e^a_{\ k} = \delta^i_{\ k}$, we can write the density (50) in the form

$$\rho = -\frac{1}{\nu c^2}T^a_{\ bk}T^b_{\ a}{}^k. \tag{52}$$

Applying now transformations defined on the rotation group $O(3.1)$ to $T^{a}_{\ bk}$ we can turn these quantities to zero. Consequently, the density (52), and hence the mass (51), are relative in nature, as follows from the principle of universal relativity.

The relativity of mass has been found experimentally in the theory of physical vacuum proposed by Dirac who created quantum electrodynamics. In quantum electrodynamics, experiments on the production of electron-positron pairs from vacuum when vacuum absorbed γ-quanta with energy $E \geq 2m_o c^2$, where m_o is the electron rest mass, have shown that the rest mass m_o is a relative quantity. Till the pair production, the rest mass of the system was zero, since there existed only γ-quanta with zero rest mass. After the production we have an electron and a positron, both with a non-zero rest mass m_0, so that in the system we now have a rest mass $2m_0$.

4 Field model of a point particle

Let us examine the solution of the vacuum set of the Einstein-Yang-Mills equations $(A), (B.1)$ and $(B.2)$, which describes the spherically symmetrical formation produced from vacuum. This enables us to establish the correspondence of the equations of physical vacuum to the fundamental equations of field theory, which yield a point, spherically symmetrical model of a particle. Indeed, the only model of a particle that follows from the fundamental equations of modern field theory is the model of a point particle with the matter density proportional to the Dirac δ-function

$$\rho \sim \delta(\mathbf{r}). \tag{53}$$

Static solutions of field equations in Newton's gravitation theory and the Maxwell-Lorentz electrodynamics with a point source on their right-hand side yield the Coulomb-Newton interaction potential

$$\varphi \sim \frac{\alpha}{r}. \tag{54}$$

We will consider the spherically symmetrical solution of the vacuum equations (A) and (B), which describe the vacuum excitation with a variable Coulomb-Newton potential and for which the Energy-momentum tensor (5) is different from zero. This solution has the following characteristics [1]:

Solution with a variable Coulomb-Newton potential

(55)

1. Coordinates $x^0 = u, x^1 = r, x^2 = \theta, x^3 = \varphi$.
2. Components of the Newman-Penrose symbols

$$\sigma^{i}_{0\dot{0}} = (0,1,0,0), \quad \sigma^{i}_{1\dot{1}} = (1,U,0,0), \quad \sigma^{i}_{0\dot{1}} = \rho(0,0,P,iP),$$

$$\sigma_{i}^{0\dot{0}} = (1,0,0,0), \quad \sigma_{i}^{1\dot{1}} = (-U,1,0,0), \quad \sigma_{i}^{0\dot{1}} = -\frac{1}{2\rho P}(0,0,1,i)$$

$$U(u) = -1/2 + \Psi^0(u)/r, \quad P = (2)^{-1/2}(1 + \zeta\bar{\zeta}/4), \quad \zeta = x^2 + ix^3,$$
$$\Psi^0 = \Psi^0(u).$$

3. Spinor components of the torsion field

$$\rho = -1/r, \quad \alpha = -\overline{\beta} = -\alpha^0/r, \quad \gamma = \Psi^0(u)/2r^2,$$

$$\mu = -1/2r + \Psi^0(u)/r^2, \quad \alpha^0 = \zeta/4.$$

4. Spinor components of the Riemann tensor

$$\Psi_2 = \Psi = -\Psi^0(u)/r^3, \quad \Phi_{22} = \Phi = -\dot{\Psi}^0(u)/r^2 = -\frac{\partial \Psi^0}{\partial u}\frac{1}{r^2}.$$

The Riemann metric of the solution (55) in (quasi) spherical coordinates has the form

$$ds^2 = \left(1 - \frac{2\Psi^0(t)}{r}\right)c^2dt^2 - \left(1 - \frac{2\Psi^0(t)}{r}\right)^{-1}dr^2 - -r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{56}$$

Using the solution (55), we can determine the explicit form of the energy-momentum tensor (6). Calculations will show that the tensor is

$$T_{jm} = \rho c^2 l_j l_m, \tag{57}$$

where ρ is the matter density of a vacuum excitation given by

$$\rho = -\frac{2\dot{\Psi}^o(u)}{\nu c^2 r^2}, \quad \dot{\Psi}^o(u) < 0\ ; \tag{58}$$

$l_m l^m = 0$ is the light like vector, which is the spinor basis of the solution (55).

We now consider the limiting process $\Psi^o(u) \to \Psi^o = const$ of the matter density in the solution (55). We introduce the auxiliary parameter ξ with the dimensionality of length

$$\xi = \frac{\pi|\dot{\Psi}^o|r^2}{2\Psi^o}. \tag{59}$$

Through the parameter ξ the density module (58) can be represented as

$$\rho = \rho^+ = \frac{8\pi\Psi^o}{\nu c^2}\frac{1}{2\pi r^2}\frac{\xi}{r^2} = \frac{8\pi\Psi^o}{\nu c^2}\frac{1}{2\pi r^2}\frac{\xi}{(r^2 + \xi^2)}\left(1 + \frac{\xi^2}{r^2}\right), \tag{60}$$

where the $+$ sign implies that the density ρ^+ defines right-hand matter with a positive density and positive mass. Taking the limit in (60) for $\xi \to 0$, i.e., for $\Psi^o(u) \to \Psi^o = const$, and using the well-known formula

$$\frac{1}{2\pi r^2}\frac{1}{\pi}\lim_{x \to 0}\left(\frac{x}{x^2 + r^2}\right) = \frac{1}{2\pi r^2}\delta(r) = \delta(\mathbf{r}),$$

where $\delta(\mathbf{r})$ is the three-dimensional Dirac function, we will get

$$\rho^{+} = \frac{8\pi\Psi^{o}}{\nu c^{2}}\frac{1}{2\pi r^{2}}\delta(r) = \frac{8\pi\Psi^{o}}{\nu c^{2}}\delta(\mathbf{r}). \tag{61}$$

It is seen from this relationship that when a vacuum excitation becomes stationary the matter density distributed over space **coincides with the matter density for a point particle** (Dirac's δ-function describes the distribution of a point source).

The result obtained substantiates Einstein's assumption that in a purely field theory a point particle must appear as a limiting case and not be introduced into the theory in an artificial manner, since: "... a combination of the idea of a continuous field with concepts of material points located discretely in space turns out to be contradictory. A consistent field theory requires that all the elements be continuous not only in time but also in space, and at all its points at that. Consequently, the material point is out of place in field theory [2]."

The fact that a material point appears in a purely field theory as a limiting stationary case is one of the most important results of the theory of physical vacuum.

5 Real point massive particles produced from vacuum

We will now consider the correspondence of the equations of physical vacuum (A) and (B) to those fundamental equations of modern physics in which particles are point and stable particles. If we take into account the above results, the equation $(B.1)$ in the set **(??)** for a point stationary source can be written as

$$R_{jm} - \frac{1}{2}g_{jm}R = \nu T_{jm}, \tag{62}$$

where

$$T_{jm} = \frac{8\pi\Psi^{0}}{\nu c^{2}}\delta(\mathbf{r})l_{j}l_{m}. \tag{63}$$

Now we compare equations (62) with Einstein's equations describing a point source. We note that these equations coincide when in the relationship (63) before the δ-function stands the mass of the point source, i.e.,

$$M = \frac{8\pi\Psi^{0}}{\nu c^{2}}. \tag{64}$$

On the other hand, as the source goes stationary, the metric (56) becomes the Schwarzshield metric (i.e., the solution of Einstein's equations for a point source) provided that

$$\Psi^{0} = \frac{MG}{c^{2}}. \tag{65}$$

Substituting (65) into the equality (64), we will obtain the value of initially arbitrary factor ν in the vacuum equations (62)

$$\nu = \frac{8\pi G}{c^{4}}. \tag{66}$$

In that case the equations (62) completely coincide with Einstein's equations that describe the gravitational field of a point source with constant mass. Accordingly, the metric (56) becomes the Schwarzshield metric

$$ds^2 = (1 - \frac{2MG}{rc^2})c^2dt^2 - (1 - \frac{2MG}{rc^2})^{-1}dr^2 - \\ -r^2(d\theta^2 + \sin^2\theta d\varphi^2). \qquad (67)$$

The equations of motion for a test particle in the theory of physical vacuum coincide with the equations of geodesics of the space of absolute parallelism

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} + T^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \qquad (68)$$

To clarify the correspondence of the equations of physical vacuum and Einstein's theory we will use the equations (68). These equations coincide with the equations of motion in Einstein's gravitation theory on the condition that

$$F^i_I = mT^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0,$$

i.e., if the inertia forces are zero. This means that the equations of motion in Einstein's theory

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0 \qquad (69)$$

are written either in inertial or locally inertial reference frames.

To show this we will write for simplicity the solution (55) with a constant source function $\Psi^0 = MG/c^2 = const$ in conventional quasi-Cartesian coordinates. In these coordinates the tetrad $e^a{}_i$ becomes

$$e^{(0)}{}_0 = (1 + \frac{2\varphi}{c^2})^{1/2}, \quad e^{(1)}{}_1 = e^{(2)}{}_2 = e^{(3)}{}_3 = (1 - \frac{2\varphi}{c^2})^{1/2}, \qquad (70)$$

where in parentheses we have the tetrad indices and $\varphi = -MG/r$.

The Riemannian metric for the tetrad (70) can be derived using the relationships

$$g_{ik} = \eta_{ab}e^a{}_ie^b{}_k, \quad \eta_{ab} = \eta^{ab} = \mathrm{diag}(1 - 1 - 1 - 1).$$

It can be written as

$$ds^2 = (1 - \frac{2MG}{rc^2})c^2dt^2 - (1 + \frac{2MG}{rc^2})(dx^2 + dy^2 + dz^2). \qquad (71)$$

If now we consider the nonrelativistic approximation and take gravitational fields to be weak, i.e., assume that

$$\frac{2\varphi}{c^2} \ll 1, \quad g_{ik} \simeq \eta_{ik}, \quad ds \simeq ds_0 = cdt(1 - \frac{v^2}{c^2})^{1/2}, \qquad (72)$$

$$R^i{}_{jkm} \simeq \overset{o}{R}{}^i{}_{jkm} = 0, \quad \frac{v^2}{c^2} \ll 1, \quad ds \simeq ds_0 \simeq cdt,$$

we will derive from (69) the following equations of motion of a mass m

$$m\frac{d^2x^\alpha}{dt^2} = -mc^2\Gamma^\alpha{}_{00} = m\frac{MG}{r^3}x^\alpha. \tag{73}$$

It is easily seen that the equations derived are the equations of motion of the Newtonian theory of gravitation, which are known to be written in inertial reference frames. Note that in passing from Einstein's equations of motion (69) to the Newtonian equations of motion (73) we have never used transformations of the coordinates x^i. Therefore, transitions from accelerated reference frames to inertial ones are absolutely out of the question. This suggests that both (73) and (69) are written for inertial reference frames, although they allow coordinate transformations that correspond to a transition to a locally inertial reference frame. In transition to an accelerated locally Lorentzian reference frame the equations (69) take the form $d^2x/ds^2 = 0$. These are the equations of free motion.

A consistent description of the transition to a locally Lorentzian reference frame is only possible using the equations (68). Indeed, in passing to an accelerated reference frame in Newton's equations (73) an inertia force should appear. It is this situation that is described by the equations (68). For the conditions (72) the equations of motion (68) for a mass will be written as

$$m\frac{d^2x^\alpha}{dt^2} = m\frac{MG}{r^3}x^\alpha - m\frac{MG}{r^3}x^\alpha = 0. \tag{74}$$

Here $F^\alpha_I = -mc^2T^\alpha_{00} = -mMGx^\alpha/r^3$ is the inertia force that compensates for the locally gravitational force $F^\alpha_G = mMGx^\alpha/r^3$. It is owing to this compensation that a local weightlessness condition in an accelerated locally Lorentzian frame is produced.

To conclude, we can say that the equations of physical vacuum describe stable point gravitating particles produced from vacuum and satisfying the equations in Einstein's theory of gravitation provided that:

6 λ-term and dark energy

When spherically symmetrical massive matter becomes stationary, the field equations ($B.1$) and the energy-momentum tensor (47) in an inertial reference frame become

$$R_{jm} - \frac{1}{2}g_{jm}R = \frac{8\pi G}{4}T_{jm}, \tag{75}$$

where

$$T_{jm} = -Mc^2\delta(\mathbf{r})(u_j u_m - \frac{1}{2}g_{jm}). \tag{76}$$

Equations (75) with the matter tensor (76) can be rewritten as

$$R_{jm} - \frac{1}{2}g_{jm}R + \lambda g_{jm} = -\frac{8\pi G}{c^4}T^{(d)}_{jm}, \tag{77}$$

where

$$T^{(d)}_{jm} = Mc^2\delta(\mathbf{r})u_j u_m \tag{78}$$

is the "dust" tensor and

$$\lambda = -\frac{4\pi GM}{c^2}\delta(\mathbf{r}) \tag{79}$$

is sam kind of λ-term.

Equations (77) are interesting in that they enable a "point" particle to be modeled as a microscopic black hole and the approximate solutions of the vacuum equations to be sought within the framework of Einstein's theory beyond the source (when $2MG/rc^2 \ll 1$), near the horizon of the black hole (when $2MG/rc^2 \approx 1$) and inside the source ($2MG/rc^2 > 1$). Corresponding to these three cases are the following equations:

$$R_{jm} = 0, \qquad 2MG/rc^2 \ll 1, \tag{80}$$

$$R_{jm} - \frac{1}{2}g_{jm}R = -\frac{8\pi G}{c^4}T^{(d)}_{jm}, \qquad 2MG/rc^2 \approx 1, \tag{81}$$

$$R_{jm} - \frac{1}{2}g_{jm}R + \lambda g_{jm} = -\frac{8\pi G}{c^4}T^{(d)}_{jm}, \qquad 2MG/rc^2 > 1. \tag{82}$$

Equations (81), which in Einstein's theory are fundamental and describing a point source, in the theory of physical vacuum are approximate equations that are valid near the horizon. Exact correspondence of the equations of physical vacuum to the field equations in Einstein's theory of gravitation only holds for the vacuum equations (80).

Considering that $g_{jm}g^{jm} = 4$, we can rewrite the tensor (47) as

$$T_{jm} = \frac{1}{\nu}\varphi^2(x^i)(\frac{1}{4}g_{jm}g^{jm}u_j u_m - \frac{1}{2}g_{jm}) = \tag{83}$$
$$= \frac{1}{\nu}\varphi^2(x^i)(\frac{1}{4}g_{jm} - \frac{1}{2}g_{jm}) = -\frac{1}{4\nu}\varphi^2(x^i)g_{jm},$$

or

$$T_{jm} = \lambda(x^i)g_{jm}, \tag{84}$$

where

$$\lambda(x^i) = -\frac{1}{4\nu}\varphi^2(x^i) = \frac{\rho c^2}{4}.$$

In this case λ-term is generated by a scalar field φ which has torsional nature and also is a source of a "dark matter".

References

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